INTERPENETRATION OF TWO IONIZED GAS CLOUDS

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ABSTRACT. Collisions of two ionized gas clouds have been considered and it is shown that the counter streaming will, in general, be unstable and the double stream will form small clouds of space charge at the expense of streaming energy of the clouds. These small clouds are responsible for the geomagnetic storms and for the generation of radio noise from colliding galaxies.

INTRODUCTION

Collision of two fully ionized gas clouds is of considerable importance in explaining the geomagnetic storms and the generation of radio noise from the colliding galaxies. It is found that the counter streaming of gas cloud will in general be unstable except for the case when the density of one of the streams is extremely low as compared to that of the other. The double stream will form small clouds of space charge at the expense of the streaming energy of the clouds. These small clouds may be able to reach the earth and diffuse through the geomagnetic field producing geomagnetic storm. These space charge clouds, because of their stray electric fields, may be responsible for the generation of radio noise from the colliding galaxies.

COUNTERSTREAMING OF IONIZED GAS CLOUDS

Suppose that a completely ionized neutral gas cloud of initial uniform density N_{o1} electrons and N_{o1} protons per cm^3 moving with initial uniform velocity $\mathbf{V}_{o1}$. Let a similar stream with density N_{o2} electrons and N_{o2} protons per cm^3 be moving with velocity $-\mathbf{V}_{o2}$. We shall further assume that the temperature and velocity of each stream be such that the thermal motion among the particles and the collisions by coulomb interaction may be ignored.

Let us consider that these two gas clouds impinge upon one another. After the counterstreaming we shall assume the deviation in the densities and velocities from the initial uniform streaming to be of the first order only and are regarded as functions of position and time. We shall use the subscripts 1 and 2 to refer to the two streams and subscripts e and p to denote the electrons and protons respectively. Thus we let their densities and velocities be

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$$N_{e1} = N_{o1}+n_{e1}; \qquad N_{p1} = N_{o1}+n_{p1} \quad \ldots \quad (1)$$

$$N_{e2} = N_{o2}+n_{e2}; \qquad N_{p2} = N_{o2}+n_{p2} \quad \ldots \quad (2)$$

$$\mathbf{V}_{e1} = \mathbf{V}_{o1}+\mathbf{v}_{e1}; \qquad \mathbf{V}_{p1} = \mathbf{V}_{o1}+\mathbf{v}_{p1} \quad \ldots \quad (3)$$

$$\mathbf{V}_{e2} = -\mathbf{V}_{o2}+\mathbf{v}_{e2}; \qquad \mathbf{V}_{p2} = -\mathbf{V}_{o2}+\mathbf{v}_{p2} \quad \ldots \quad (4)$$

Then if the electric field is E, we have the Poisson equation

$$\text{div } E = 4\pi e(n_{e1}-n_{p1}+n_{e2}-n_{p2}) \quad \ldots \quad (5)$$

The equations of motion are

$$\left[\frac{\partial}{\partial t}+(\mathbf{V}_{e1}\cdot \text{grad})\right] \mathbf{V}_{e1} = \frac{e}{m}\mathbf{E} \quad \ldots \quad (6)$$

$$\left[\frac{\partial}{\partial t}+(\mathbf{V}_{p1}\cdot \text{grad})\right] \mathbf{V}_{p1} = -\frac{e}{M}\mathbf{E} \quad \ldots \quad (7)$$

$$\left[\frac{\partial}{\partial t}+(\mathbf{V}_{e2}\cdot \text{grad})\right] \mathbf{V}_{e2} = \frac{e}{m}\mathbf{E} \quad \ldots \quad (8)$$

$$\left[\frac{\partial}{\partial t}+(\mathbf{V}_{p2}\cdot \text{grad})\right] \mathbf{V}_{p2} = -\frac{e}{M}\mathbf{E} \quad \ldots \quad (9)$$

and the equations of continuity are

$$\left[\frac{\partial}{\partial t}+(\mathbf{V}_{e1}\cdot \text{grad})\right] N_{e1}+N_{e1} \text{ div } \mathbf{V}_{e1} = 0 \quad \ldots \quad (10)$$

$$\left[\frac{\partial}{\partial t}+(\mathbf{V}_{p1}\cdot \text{grad})\right] N_{p1}+N_{p1} \text{ div } \mathbf{V}_{p1} = 0 \quad \ldots \quad (11)$$

$$\left[\frac{\partial}{\partial t}+(\mathbf{V}_{e2}\cdot \text{grad})\right] N_{e2}+N_{e2} \text{ div } \mathbf{V}_{e2} = 0 \quad \ldots \quad (12)$$

$$\left[\frac{\partial}{\partial t}+(\mathbf{V}_{p2}\cdot \text{grad})\right] N_{p2}+N_{p2} \text{ div } {}_{p}\mathbf{V}_{2} = 0 \quad \ldots \quad (13)$$

Here e and m are the electronic charge and mass respectively and $-e$ and M are the respective quantities for the protons. Combining equations of the motion (Eqns. (6) to (9)) and equations of continuity (Eqns. (10) to (12)) and using Eqns (1) to (5) we get after some simplification

$$\left[\frac{\partial}{\partial t}+(\mathbf{V}_{o1}\cdot \text{grad})\right]^2 n_{e1} = -\omega^2_{e1}(n_{e1}-n_{p1}+n_{e2}-n_{p2}) \quad \ldots \quad (14)$$

$$\left[\frac{\partial}{\partial t}+(\mathbf{V}_{o1}\cdot \text{grad})\right]^2 n_{p1} = \omega^2_{p1}(n_{e1}-n_{p1}+n_{e2}-n_{p2}) \quad \ldots \quad (15)$$

$$\left[\frac{\partial}{\partial t}-(\mathbf{V}_{o2}\cdot \text{grad})\right]^2 n_{e2} = -\omega^2_{e2}(n_{e1}-n_{p1}+n_{e2}-n_{p2}) \quad \dots \quad (16)$$

$$\left[\frac{\partial}{\partial t}-(\mathbf{V}_{o2}\cdot \text{grad})\right]^2 n_{p2} = \omega^2_{p2}(n_{e1}-n_{p1}+n_{e2}-n_{p2}) \quad \dots \quad (17)$$

Here ω's represent the electron or proton plasma frequencies in the two streams, viz,

$$\omega_e^2 = \frac{4\pi N_o e^2}{m} \quad \text{and} \quad \omega_p^2 = \frac{4\pi N_o e^2}{M} \quad \dots \quad (18)$$

Substracting Eq. (15) from equation (14) and Eq. (17) from Eq. (16) and putting

$$a_1 = n_{e1}-n_{p1} \quad \text{and} \quad a_2 = n_{e2}-n_{p2} \quad \dots \quad (19)$$

$$\omega_1^2 = \omega_{e1}^2+\omega_{p1}^2 \quad \text{and} \quad \omega_2^2 = \omega_{e2}^2+\omega_{p2}^2 \quad \dots \quad (20)$$

We find

$$\left[\frac{\partial}{\partial t}+(\mathbf{V}_{o1}\cdot \text{grad})\right]^2 a_1 = -\omega_1^2(a_1+a_2) \quad \dots \quad (21)$$

$$\left[\frac{\partial}{\partial t}-(\mathbf{V}_{o2}\cdot \text{grad})\right]^2 a_2 = -\omega_2^2(a_1+a_2) \quad \dots \quad (22)$$

The solutions involving a's can be represented as

$$a = a_o \exp i(\sigma t+\vec{\kappa}\cdot \mathbf{r}) \quad \dots \quad (23)$$

here $\mathbf{r}$ is the space vector.

In the solution of the above type, with a given real $\vec{\kappa}$, the counterstreaming will be unstable for which σ is a complex quantity. In this case the amplitude of the corresponding oscillations can grow indifinitely. The minimum value of wave number κ_{min} for which this is possible gives the maximum value of the wave length λ_{max} at which there is instability. The double stream will then be unstable for complex value of σ and will form small clouds of space charge with maximum length λ_{max} at the expense of the streaming energy of the beam.

To determine the dispersion relation between σ and κ we substitute equation (23) into Eqns. (21) and (22) and after rearrangement we get

$$[\{\sigma+(\mathbf{V}_{o1}\cdot\vec{\kappa})\}-\omega_1^2]a_{o1} = \omega_1^2 a_{o2} \quad \dots \quad (24)$$

and

$$[\{\sigma-(\mathbf{V}_{o2}\cdot\vec{\kappa})\}-\omega_2^2]a_{o2} = \omega_2^2 a_{o1} \quad \dots \quad (25)$$

Elimination of a_{o1} and a_{o2} from Eqns. (24) and (25) gives the relations between σ and κ, viz..

$$\{\sigma+(\mathbf{V}_{o1}\,.\,\vec{\kappa})\}^2\{\sigma-(\mathbf{V}_{o2}\,.\,\vec{\kappa})\}^2-\omega_1^2\{\sigma-(\mathbf{V}_{o2}\,.\,\vec{\kappa})\}^2$$
$$-\omega_2^2\{\sigma+(\mathbf{V}_{o1}\,.\,\vec{\kappa})\}^2=0 \quad \ldots \quad (26)$$

DISPERSION RELATIONS

To study the nature of σ for all real values of κ we substitute

$$\left.\begin{aligned}\mathbf{V}_{o1}+\mathbf{V}_{o2}&=2U\\ \mathbf{V}_{o1}-\mathbf{V}_{o2}&=2V\end{aligned}\right\} \quad (27)$$

$$p=\omega+\mathbf{V}\,.\,\vec{\kappa} \quad \ldots \quad (28)$$

and

$$\Omega=\mathbf{U}\,.\,\vec{\kappa} \quad \ldots \quad (29)$$

in dispersion relation (26) and obtain

$$\{p^2-\Omega^2\}^2-(\omega_1^2+\omega_2^2)(p^2+\Omega^2)+2(\omega_1^2-\omega_2^2)p\Omega=0 \quad \ldots \quad (30)$$

Further, let us put

$$N_{o2}=bN_{o1} \quad \ldots \quad (31)$$

Thus we have

$$\omega_2^2=b\omega_1^2$$
$$=b\omega^2\ (\text{say}) \quad \ldots \quad (32)$$

and Eq. (30) reduces to

$$(p^2-\Omega^2)^2-(1+b)\omega^2(p^2+\Omega^2)+2(1-b)\omega^2p\Omega=0 \quad \ldots \quad (33)$$

This is the most general relation and difficult to solve exactly. We shall therefore study Eq. (33) for some specific cases of astrophysical interest.

Case (i) $b\simeq 1$. Let us first consider that the streams have initially the same densities. Thus for $h=1$ and Eq. (33) reduces to

$$(p^2-\Omega^2)^2-2\omega^2(p^2+\Omega^2)=0$$

or

$$p^4-2p^2(\Omega^2+\omega^2)+\Omega^2(\Omega^2-2\omega^2)=0 \quad \ldots \quad (34)$$

It can readily be shown that for all real values of $\vec{\kappa}$ and for

$$\Omega^2>2\omega^2 \quad \ldots \quad (35)$$

p^2 is always real and positive. However, there exists a real and negative value for p^2 provided

$$\Omega^2 < 2\omega^2 \qquad \dots (36)$$

Let this value of p^2 be $-q^2$. Thus we have

$$\sigma = -\vec{V}.\vec{\kappa}+iq \qquad \dots (37)$$

and hence for such a value of σ it is evident that the oscillations will go on build-ing up with the time and will therefore make the whole system unstable. The minimum value of $\vec{\kappa}$, $\vec{\kappa}_{min}$ for such system may be evaluated from

$$\vec{\kappa}_{min}.\,U = \sqrt{\frac{8\pi N_o e^2(m+M)}{mM}}. \qquad \dots (38)$$

and

$$\lambda_{max} = \frac{2\pi}{\kappa_{min}} \qquad \dots (39)$$

Kahn (1957) has discussed a particular case in which the two streams of equal density are moving in opposite direction having equal velocity U along the x-axis. For this case we obtain

$$\lambda_{max} = \sqrt{\frac{\pi mM}{2N_o e^2(m+M)}} \qquad \dots (40)$$

Our results are somewhat different from those of Kahn (1957) because of his assuming only the electrons interactions. The protons were assumed to provide a uniform background of positive charge because of their heavier mass, which in our opinion may not be a valid assumption.

Let us further assume that one of the streams is stationary, say $\mathbf{V}_{oz} = 0$. This gives

$$\mathbf{U} = \mathbf{V} \qquad (41)$$

and hence the stream will again be unstable and the maximum stable length will be given by equation (39) in conjunction with equation (38).

Case (ii) $b \approx 0$. Let us assume that a gas cloud is impinging into vacuum. Thus for $b \approx 0$ we get from Eq. (33) after simplification

$$(p^2-\Omega^2)^2-\omega^2(p-\Omega)^2 = 0 \qquad \dots (42)$$

It is evident from Eq. (42) that p and hence σ is always real for all real values of κ. Therefore the stream will plunge into vacuum without having any instability.

When $b >> 1$ the situation is similar to that discussed for case (ii). For $0 \leqslant b \leqslant 1$ the problem cannot be solved explicitly. The above discussion and the inspection of Eq. (33) reveals that the beam will in general be unstable except for $b \approx 0$ and for $b >> 1$

DISCUSSION OF RESULTS

It has been shown above that two penetrating streams will, in general, be unstable even when one of the streams is at rest. This is of great significance in regard to the magnetic storm theory, where the solar ion streams emanating from the disturbed solar regions produce geomagnetic storms on reaching the earth. These solar-ion streams while penetrating the solar atmosphere will become unstable and will form small clouds of space charge moving towards the earth. These clouds on entering the geomagnetic field may diffuse into the earth's magnetic field and a belt of trapped particles may be produced within the geomagnetic field, which may be responsible for the main phase of the geomagnetic storm. Kahn (1957) has alternatively suggested that the counter-streaming will be stopped because of this instability. Such an effect would prevent the passage of solar-ion stream through the solar atmosphere and inter-planetary matter. Thus, in our opinion, such an interpretation of instability may not be probable.

This instability and the formation of small clouds of space charge may also explain the generation of radio noise from the colliding galaxies e.g. Cygnus A, NGC 5128 and NGC 1275. These space charge clouds will produce the stray electric fields and the charged particles moving under the influence of this field may be responsible for the radio emission.

REFERENCE

Kahn, F. D., 1957, *J. Fluid Mech.*, 2, 601.